Name: _____ Date: _____

Algebra 1CC Ms. Moser

AVERAGE RATE OF CHANGE

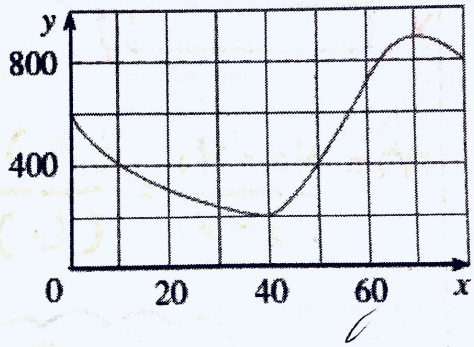
The **average rate of change** of the function $f(x)$ on the interval $x=a$ to $x=b$ is:

$$\text{Average Rate of Change} = \frac{f(b)-f(a)}{b-a}$$

$$\text{SLOPE} = \frac{y_2-y_1}{x_2-x_1}$$

The average rate of change is the SLOPE of the line between the values of *a and b* on the graph of *f(x)*, that is, the line that passes through the points *(a, f(a)) and (b, f(b))*.

Finding the average rate of change is the same as finding the SLOPE of a line.



AVERAGE RATE OF CHANGE

For the function $y = f(x)$, the average rate that $f(x)$ changes from $x = a$ to $x = b$ is given by:

$$\frac{f(b)-f(a)}{b-a} = \frac{\text{how much the y-values have changed}}{\text{how much the x-values have changed}}$$

Examples: Find the average rate of change on the indicated intervals.

I do...

1) Consider the function given by $f(x) = x^2 + 3$. Find its average rate of change from $x = -1$ to $x = 3$.

put in calc $Y=$

① Find Y-values (Look on table)

	X	Y	
x_1	-1	4	y_1
x_2	3	12	y_2

② Plug in to Slope formula

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{(12)-(4)}{(3)-(-1)} = 2$$

$m = 2$

You do...

2) Consider the function $f(x) = \frac{x-1}{x+2}$. Find its average rate of change on $[-1, 3]$.

$Y=$ $\frac{x-1}{x+2}$ X_1 X_2

① Find Y-values on calc table

	X	Y	
x_1	-1	-2	y_1
x_2	3	$\frac{2}{5}$	y_2

② Plug in to slope formula.

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{\left(\frac{2}{5}\right)-(-2)}{(3)-(-1)} = \frac{3}{5}$$

I do...

3) The function $h(x)$ is given in the table below. Find the average rate of change over the interval $2 \le x \le 6$?

use x-values 2 and 6

x	$h(x)$
0	10
2	9
4	6
6	3

x_1 y_1 x_2 y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(3)-(9)}{(6)-(2)} = -\frac{3}{2}$$

$m = -\frac{3}{2}$

You do...

4) The function $t(x)$ is given in the table below. Find the average rate of change from 3 years to 5 years?

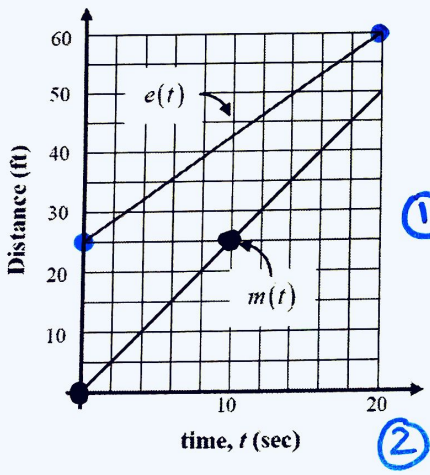
Time (years)	1	2	3	4	5
Height (in.)	27	35	37	42	45

x_1 x_2 y_1 y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(45)-(37)}{(5)-(3)} = 4$$

$m = 4$ inches per year.

5) Max and his younger sister Evie are having a race in the backyard. Max gives his sister a head start and they run for 20 seconds. The distance they run, in feet, is given below with Max's distance given by the function $m(t)$ and Evie's distance given by the function $e(t)$.



a) Find Max's speed (average rate of change). In other words how many feet per second does he run? Express your answers as decimals and attach units. ↳ "Rate of Speed"

① Find 2 points on $m(t)$ graph.

(0,0) (10, 25)
x_1 y_1 x_2 y_2

② Plug in to slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(25)-(0)}{(10)-(0)} =$$ 2.5 ft per second.

b) Find Evie's speed (average rate of change).

(0, 25) (20, 60)
x_1 y_1 x_2 y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(60)-(25)}{(20)-(0)} =$$ 1.75 ft per second

YOU DO! Find the rate of change of each of the following functions! Show all work!

6) Frances is selling glasses of lemonade. The function $g(t) = \frac{t^2+4}{2}$ models the number of glasses she had sold, g, after t-hours. What is the average rate at which she is selling lemonade between $t = 2$ and $t = 6$ hours?

using t instead of x.

x	y
x_1 2	4 y_1
x_2 6	20 y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(20)-(4)}{(6)-(2)} = 4 \text{ glasses per hour.}$$

7) The function $f(x)$ is given in the table below. Find its average rate of change between the following points. Show the calculations that lead to your answer.

$x = -3$ **to** $x = 1$

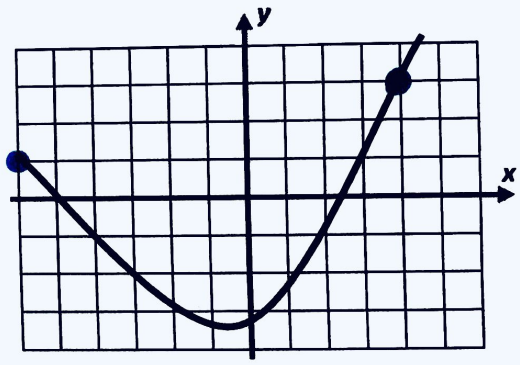
	x	$f(x)$	
x_1	−3	7	y_1
	0	−2	
x_2	1	3	y_2
	4	−8	

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(3)-(7)}{(1)-(-3)} = -1$$

8) The function $f(x)$ is given in the graph below. Find its average rate of change between the following points. Show the calculations that lead to your answer.

$x = -6$ to $x = 4$

$(-6, 1)$ $(4, 3)$



$$m = \frac{3-1}{4-(-6)} = \frac{1}{5}$$

9) The following table shows the number of points the Arlington girls team scored in their last basketball game where t is the time passed in minutes and $f(t)$ the total number of points scored after t minutes.

t	$f(t)$
0	0
8	30
16	48
24	55
32	64

First Half

(a) What was the average rate they were shooting in the first half of the game? Be sure to include proper units in your answer.

$$m = \frac{48-0}{16-0} = 4 \text{ points per minute.}$$

(b) What was their average rate over the whole game?

$$\frac{64-0}{32-0} = 2 \text{ points per minute.}$$

(c) Given your answers above which half of the game do you feel they had a better rate of scoring?

First half, they had a higher scoring rate.