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Linear Versus Exponential
Common Core Algebra I

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Linear and exponential functions share many characteristics. This is because they are based on two different, but similar, sets of principles.

Linear Versus Exponential

Linear functions are based on **repeatedly adding** the same amount (the slope). + −

Exponential functions are based on **repeatedly multiplying** by the same amount (the base). × ÷ %

***Exercise* #1:** The two tables below represent a linear function and an exponential function. Which is which? Explain how you arrive at your answer.

Table 1

x	0	1	2	3	4
y	5	10	20	40	80

×2 ×2 ×2 ×2

Exponential, repeated multiplication.

Table 2

x	0	1	2	3	4
y	8	11	14	17	20

+3 +3 +3 +3

Linear, repeated addition.

***Exercise* #2:** Find equations in standard form for each of the functions from *Exercise* #1.

(a) Table 1 $y=a(b)^x$

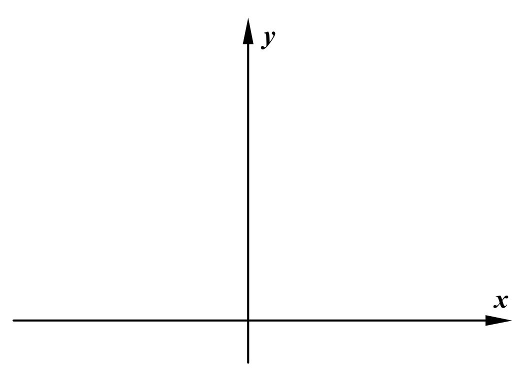
$y=5(2)^x$

(b) Table 2 $y=mx+b$

$y=3x+8$

It is interesting that linear and exponential functions are ones where two points on the curve will always determine the equation of the curve.

***Exercise* #3:** Consider the two points $(0,12)$ and $(1,3)$. Create a linear equation that passes through these points in $y=mx+b$ form and an exponential equation in $y=a(b)^x$ form that also passes through them. Then, using your calculator, graph both using a **window** of $-2\le x\le 2$ and $-5\le y\le 15$.



Recall that linear functions have a constant **average rate of change (slope)**. That's, of course, why they have a constant amount added for every constant change in x. Let's examine the average rate of change for an increasing exponential.

***Exercise* #4:** The exponential function $f(x)=4(2)^x$ is shown partially in the table below. Find the average rate of change over the various intervals given. This should be relatively simple because $\Delta x=1$.

x	0	1	2	3	4
y	4	8	16	32	64

×2 ×2 ×2

$y=a(b)^x$

$\boxed{y=4(2)^x}$

(a) $0\le x\le 1$

(b) $1\le x\le 2$

(c) $2\le x\le 3$

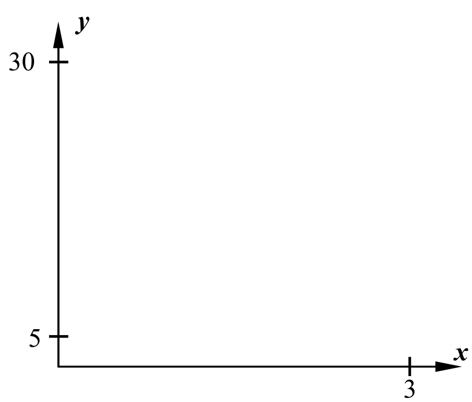
(d) $3\le x\le 4$

(e) What is clearly happening to the average rate of change as x gets larger?

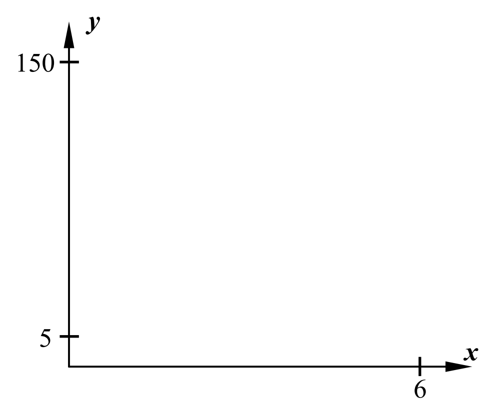
The fact that the **slope** of an **increasing exponential** is **always increasing** has an interesting consequence.

***Exercise* #5:** Consider the linear function $y=20x+5$ and the exponential function $y=5(2)^x$. Both of these functions have a y-intercept of 5, so "start" in the same location.

(a) Using your calculator, sketch these two curves on the axes below for the indicated window. Label each with its equation.



(b) Again, using your calculator, sketch these two curves on the axes below for the indicated window. Label each with its equation.



(c) Although the line appears to rise more quickly than the exponential, at first, the exponential eventually catches up and surpasses the linear. Why will an **increasing exponential function** always catch up with an **increasing linear function**?

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Linear Versus Exponential
Common Core Algebra I Homework

Fluency

1. For each of the following problems a table of values is given where $\Delta x = 1$. For each, first determine if the table represents a linear function, of the form $y = mx + b$, or an exponential function, of the form $y = a(b)^x$. Then, write its equation.

(a)

x	-1	0	1	2	3
y	4	7	10	13	16

+3 +3 +3 +3

* y-int is where $x=0$

Type: Linear $y=mx+b$

Equation: $y=3x+7$

(b)

x	0	1	2	3	4
y	2	6	18	54	162

Type: ______________

Equation: ______________

(c)

x	-2	-1	0	1	2
y	32	16	8	4	2

$\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$

$\div 2$ $\div 2$ $\div 2$ $\div 2$

Type: Exponential $y=a(b)^x$

Equation: $y=8\left(\frac{1}{2}\right)^x$

(d)

x	-2	-1	0	1	2
y	32	16	0	-16	-32

Type: ______________

Equation: ______________

(e)

x	0	1	2	3	4
y	16	20	25	$31\frac{1}{4}$	$39\frac{1}{16}$

$\frac{20}{16}=\frac{5}{4}$

$\times\frac{5}{4}$ $\times\frac{5}{4}$ $\times\frac{5}{4}$

Type: exponential $y=a(b)^x$

Equation: $y=16\left(\frac{5}{4}\right)^x$

(f)

x	0	1	2	3	4
y	180	160	140	120	100

Type: ______________

Equation: ______________

2. The data shown in the table below represents either a linear or an exponential function. Which of the equations below best models this data set?

x	1	2	3	4
y	10	20	40	80

(1) $y=5(2)^x$ $\quad$ $y=2x+10$

(2) $y=10(2)^x$ $\quad$ (4) $y=10x+5$

APPLICATIONS

3. Wildlife biologists are tracking the population of albino deer in an upstate New York forest preserve. They record the population every year since 2005, which they consider to be $t=0$. Their data is shown in the table below.

Year	2005	2006	2007	2008	2009	2010
t	0	1	2	3	4	5
Population	86	98	111	128	147	168

(a) Although neither a linear nor an exponential function would model this data perfectly, justify why an exponential function would be a much better fit. Specifically, explain both why a linear function would *not* be a good fit while an exponential would be reasonable.

(b) Determine an equation for an exponential that models this data set in the form $P=a(b)^t$.

(c) Use your model to predict the population of deer in the year 2014.

REASONING

4. You can determine the equation of a line or the equation of an exponential given any two points that lie on these curves. In this exercise we will pick two special points. Consider the points $(0,5)$ and $(1,15)$.

(a) Write the equation of the line that passes between these two points in $y=mx+b$ form.

(b) Write the equation of the exponential that passes between these two points in $y=a(b)^x$ form.

(c) Using your calculator, sketch the two curves on the axes below. Label with their equations.