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Sequences: Lesson 4 - Recursive Sequence Formulas
Warm Up: Given the explicit formula $a_{n}=2 n+10$ for $n \geq 1$ and that $a_{1}=12$ :
(a) Find the next five terms.

(b) If we know that $a_{1}=12$, how did we go from $a_{1}$ to $a_{2}$ ? How did we get from $a_{2}$ to $a_{3}$ ? Justify.


Because 2 is the common difference.

## Learning Target "I can write and use recursive arithmetic and geometric formulas."

 A Recursive formula is a rule that allows any term of a sequence, except the first, to be computed using a previous term. You must always state_ $a_{1}$ then $a_{n}$ when writing a recursive formula.Investigation 1: Given the following 5 terms in a sequence-

$$
a_{1}=5, a_{2}=8, a_{3}=11, a_{4}=14, a_{5}=17
$$

(a) How can we find the fifth term, in terms of the fourth term?
(b) How can we find the sixth term in terms of the fifth term?
(c) How can we find the $(n+1)$ term in terms of the $n$th term?

Investigation 2: Given the following sequence $5376,1344,336, \ldots$, find the fifth term in the sequence.

There is a recursive formula for each type of sequence, as follows:

## Recursive Arithmetic

$a_{n}=a_{n-1}+d$
where $d=$ the common difference
Previous.
term

Recursive Geometric


Exercise 1. Write a recursive formula for the sequence $23, \underline{29}, \underline{35}, \underline{41}$,
Step 1: Determine if the sequence is arithmetic or geometric. Find $d\left(\boldsymbol{d}=\boldsymbol{a}_{2}-\boldsymbol{a}_{1}\right)$ or $r\left(\boldsymbol{r}=\frac{\boldsymbol{a}_{2}}{a_{1}}\right)$.

$$
\text { Arithmetic } \quad d=6
$$

Step 2: Use the recursive formula for an arithmetic sequence: $a_{n}=a_{n-1}+d$ to substitute $d$.

$$
\begin{aligned}
& a_{1}=23 \\
& a_{n}=a_{n-1}+6
\end{aligned}
$$

Exercise 2- Write a recursive formula for the sequence 6, 24, 96, ...

$$
\begin{array}{r}
2 \\
\times 4 \times 4
\end{array}
$$

Step 1: Determine if the sequence is arithmetic or geometric. Find $d$ or $r$.

$$
\text { Geometric } \quad r=4
$$

Step 2: Use the recursive formula for a geometric sequence: $a_{n}=a_{n-1}(r)$, to substitute $r$.

$$
\begin{aligned}
& a_{1}=b_{1}=(4) \\
& a_{n}=a_{n-1}(4)
\end{aligned}
$$

Problem Set:
(1) Given the following sequence $\mathbf{1 2}, \mathbf{9}, \mathbf{6}, \mathbf{3}$...
a) Write a recursive formula for the sequence

$$
\begin{aligned}
& a_{1}=12 \\
& a_{n}=a_{n-1}-3
\end{aligned}
$$

b) What is the $8^{\text {th }}$ term of the sequence?

(2) Consider a sequence given by the formula $a_{n}=\frac{P T-5}{a_{n-1}-5}$, where $a_{1}=12$.
a) List the first five terms of the sequence
(A) $d=-5$

$$
\frac{12}{2}, \frac{7}{2}, \frac{-3}{2}, \frac{-8}{2}
$$

b) Write an explicit formula

$$
\begin{aligned}
& \text { rite an explicit formula } \\
& \begin{aligned}
& a_{n}= a_{1}+d(n-1) \\
& a_{n}= 12-5(n-1) \\
& 12-5 n(1-5)=-5 n+17
\end{aligned}
\end{aligned}
$$

(3) A sunflower is 3 inches tall at week 0 and grows 2 inches each week. Which functions) shown below can be used to determine the height, $f(n)$, of the sunflower in $n$ weeks?
I. $f(n)=2 n+3$
II. $f(n)=2 n+3(n-1)$
III. $f(n)=f(n-1)+2$ where $f(0)=3$
(1) I and II
(3) III, only
(2) II, only
(4) I and III
(4) If $a_{1}=6$ and $a_{n}=3+2\left(a_{n-1}\right)^{2}$, then $a_{2}$ equals..
(1) 75
(3) 180
(2) 147
(4) 900
(5) Given the recursive formula:

$$
\begin{gathered}
a_{1}=3 \\
a_{n}=\frac{2\left(a_{n-1}+1\right)}{2(\mathrm{PI}+1)}
\end{gathered}
$$

State the values of $a_{2}, a_{3}$, and $a_{4}$ for the given recursive formula.

$$
\begin{aligned}
& a_{2}=2(3+1)=8 \\
& a_{3}=2(8+1)=18 \\
& a_{4}=2(18+1)=38
\end{aligned}
$$

(6) Write the first 4 terms of the recursive sequence given that $a_{1}=3$ and $a_{n}=a_{n-1}+4$
(7) Consider a sequence given by the formula $a_{n}=a_{n-1}(-2)$, where $a_{1}=7$.
a) List the first four terms of the sequence using the recursive formula given.
b) Write an explicit formula.

